Abstract — Power systems are highly nonlinear but we depend to a large extent on linearized methods for analysis and controller design. Aspects of small disturbance stability such as its dependence on parametric change require study of the underlying bifurcation boundaries. Control measures to alleviate problems require sensitivity of margins of stability. In this paper, optimization frameworks developed at Sydney University for computing such boundaries and margins are presented along with a method to coordinate security controls.

1. INTRODUCTION

Power systems are nonlinear systems, and should be studied with nonlinear theory. Conventional linear approaches are limited for explaining characteristics properly, but we still depend on them especially for controller design. In this paper we consider the small disturbance stability of power systems in terms of the underlying nonlinear model. Small disturbance stability of a nonlinear system refers to the ability of the system to remain close to a stable equilibrium after a small disturbance. The disturbance shall be small enough so that the system can be linearized around that equilibrium point for analysis. It is the influence of large parametric change which requires deeper analysis than linear methods allow.

Under the new deregulated electricity industry structures, power grids are becoming more stressed. Further, the stability limits must be known more accurately to enable evaluation of available transfer capability in the electricity market. Techniques for assessing power system security have become more important as the system operator is legally responsible for ensuring system security. Mathematically sustaining a system secure operation corresponds to the operating point lying within stability boundaries in a space composed of power system parameters and control variables. The boundary can be very complicated. These security characteristics include power flow feasibility limits, aperiodic and oscillatory stability conditions, maximum and minimum damping conditions, and singularity induced instability conditions. The feasibility limits of a power system shall also include device limit induced bifurcations.

This paper presents optimization frameworks for computing these instability conditions and margins of stability as well as a coordinated control algorithm to maintain system security.

2. REVIEW OF BIFURCATIONS

Nonlinear power system models take the general form of differential-algebraic equations with state variables and slowly varying parameters as given below,

\[ \frac{dx}{dt} = f(x, y, \tau) \]
\[ \frac{d\gamma}{d\tau} = g(x, y, \tau) \]

where \( x \) is a vector of dynamic state variables, e.g. angles and voltages, \( y \) is a vector of algebraic state variables, and \( \tau \) is a parameter, which can be varied slowly, such as nodal powers. For small disturbance stability studies, we assume the system parameter variation is slow enough so that the model can be linearized around some equilibrium point as,

\[ \Delta x = J_1 \Delta x \]

where

\[ J_1 = f_x - f_y y^{-1} s_x \]

provided \( \det_y \neq 0 \). Then bifurcation theory can be used to predict how the system becomes unstable as parameters vary; in particular, they describe qualitative changes of the system [1]. Saddle node, Hopf and singularity induced bifurcations contribute the most to power system small disturbance stability studies. These bifurcations are now briefly described.

A saddle node bifurcation (SNB) occurs when the system Jacobian becomes singular and other transversality conditions are satisfied. A SNB indicates the disappearance of a system equilibrium as the system parameters change slowly [1]. SNBs are associated with collapse type instabilities. One of the most studied phenomena is the dynamic voltage drop at voltage collapse.

A Hopf bifurcation (HB) occurs, again with higher order conditions, when the system Jacobian has a pair of conjugate complex eigenvalues with zero real parts and nonzero complex parts; all other eigenvalues are assumed to be located on the left hand side of the complex plane. The occurrence of a Hopf bifurcation leads the nonlinear system to oscillations after slow parameter variation from an initial equilibrium. It can either lead to a persistent oscillation or a growing oscillatory transient problem. These bifurcations are referred to as supercritical and subcritical respectively.

HBs are important for power system security, because they provide a mechanism where the system may lose its stability well before the point of collapse is reached. This may proceed from being initially an oscillation event and finally lead to system collapse.
Subcritical HBs may reduce the system security operating limits due to the introduction of unstable oscillatory behavior.

Singularity induced bifurcations (SIBs) occur when the algebraic equations (2.2) becomes singular, i.e. \( \det G = 0 \). SIBs are responsible for system unpredictable behavior (in the model), and should also be carefully analyzed and avoided. At a SIB point, the system Jacobian has infinite eigenvalues. The systems' behavior is not predictable due to the modeling limitations of the network algebraic equations, and the disconnection between the algebraic and differential behaviors of the system [3, 4].

Saddle node, Hopf and singularity induced bifurcations relate to the changes in local behavior of an equilibrium, and are called local bifurcations [5]. There are bifurcations which can affect the global dynamic stability properties of the system, and are called global bifurcations. They relate to chaos and other complex phenomena which are important for nonlinear system stability analysis and control design. A steady state stable power system may lose its local stability at local bifurcations following parametric variations. Then the system may enter into sustained oscillations at stable periodic orbits, or sustained quasiperiodic oscillations at invariant tori, or sustained chaotic oscillations at strange attractors. Detailed properties of these bifurcations can be found in [1], [4-6].

The above mentioned bifurcation and load flow feasibility limits of the power system need to be more rigorously defined as operating conditions change. A power system is a large-scale system composed of a large number of devices and controls. The limits of these devices influence the feasibility regions or steady-state stability limits. These limits can be associated with limit-induced bifurcations [6].

### 3. General Framework for Computing Bifurcation Boundaries

We briefly review some of the techniques developed by the authors and colleagues at Sydney University [2], [7-15] for calculating security boundaries based on nonlinear models of the general form

\[
\dot{x}_1 = F(x_1, x_2, y, \tau) \quad (3.1)
\]

\[
0 = G(x_1, x_2, y, \tau) \quad (3.2)
\]

where \( x_1 \) is the vector of state variables, \( x_2 \) is the vector of algebraic variables, \( y \) is the vector of specified system parameters, and \( \tau \) is a vector of variable parameters chosen to illustrate particular system security conditions. For example, \( x_1 \) may include angles, speeds and voltages from dynamic loads; \( x_2 \) may be a vector of other system voltages; \( y \) may include any parameters of generators, control units, loads and networks which can be varied in planning, tuning and control, and whose influence on the dynamic stability is to be analyzed.

To locate the power flow feasibility boundary efficiently without iterative calculation procedures, except for eigenvalue calculation, the authors in [9] proposed a novel technique based on the quadratic power flow representation - the \( \Delta \)-plane technique. This special technique calculates the power flow feasibility limits using special properties of quadratic power flow equations, and the feasibility boundary can be obtained in the hyper-plane of selected power system parameters. This gives a visualization tool for power flow limit studies and related security problems.

Computational approaches for locating the saddle node and Hopf bifurcations include continuation and direct methods. Suppose the equilibrium conditions of the nonlinear system given in equations (3.1), (3.2) are represented as the simplified expression \( F(x, y, \tau) = 0 \). Saddle node bifurcations can be located by solving the set of equations,

\[
F(x, y, \tau) = 0 \quad (3.3)
\]

\[
F'(v, w) F(w, \tau) = 0 \quad (3.4)
\]

\[
\|v\| = 1 \text{ or } \|w\| = 1 \quad (3.5)
\]

where \( v, w \in \mathbb{R}^N \) are the left and right eigenvectors of the Jacobian \( F_n \) at an equilibrium defined by (3.3). Equation (3.5) ensures a nontrivial solution. The problem can be solved with the Newton-Raphson method. Neighboring equilibrium points close to the saddle node bifurcation point can be calculated by solving the equations,

\[
F(x, y, \tau) = 0 \quad (3.6)
\]

\[
(F - \varepsilon I) \nu = 0 \quad (3.7)
\]

where \( I \) is the identity matrix of the same order as the Jacobian \( F_n \), \( \varepsilon \) is a small real number. It is evident that the bifurcation point is obtained with \( \varepsilon = 0 \) [5].

A Hopf bifurcation corresponds to a pair of conjugate pure imaginary eigenvalues crossing the imaginary axis. It can be computed by solving the equations,

\[
F(x, y, \tau) = 0 \quad (3.8)
\]

\[
J_x^T(x, y, \tau) \nu^* + \nu \nu^* = 0 \quad (3.9)
\]

\[
J_y^T(x, y, \tau) \nu^* - \nu \nu^* = 0 \quad (3.10)
\]

\[
\|\nu\| = 1 \quad (3.11)
\]

where the system Jacobian is the reduced form, \( 0 \pm j\nu \) are the eigenvalues corresponding to the Hopf bifurcation, and \( \nu = \nu^* j \nu^* \) are the corresponding left eigenvectors. The last equation ensures the nontrivial solution. Again, it can be solved with a Newton like optimization method. These two approaches for computing bifurcations belong to the class of direct methods.

The authors in [10] developed a general method, which is capable of revealing most of the small disturbance stability conditions in one optimization approach in the parameter space. Given the nonlinear system Jacobian as \( J \), the general method is based on solving the following optimization problem:

\[
\min \|F(x, y, \tau)\|
\]
parameter space optimization approach. The optimization problem can provide a comprehensive and reliable approach in power system parameter space. Let's define the boundaries in the parameter space: useful to have knowledge of the geometry of the small disturbance stability conditions in one optimization approach. The optimization method can also give solutions of power flow feasibility boundary points. By rotating the ray of search in the parameter space, the bifurcation and feasibility hyper-plane can be located based on this general method.

In power system operation and control, it is also useful to have knowledge of the geometry of the small disturbance stability boundaries and margins in the power system parameter space. Let's define the stability boundary: the set of parameters which for a given parameter space $r$. In references [6] and [16], it is assumed that $\Sigma$ is a smooth hypersurface in $R^n$. There are simple geometric interpretations for different stability boundaries such as saddle node bifurcation boundary $\Sigma_{sn}$, Hopf bifurcation boundary $\Sigma_{hb}$ and the feasibility region $\Sigma_{f}$ [16]. Based on a properly chosen system model, the general method can be used to locate these boundaries as well as the closest small disturbance stability characteristic points to the given operating point. Techniques for computing these closest distances are given in the following section.

Once a characteristic point in the bifurcation boundary in located, a special parameter continuation method can be used to trace these bifurcation boundaries in the parameter space [11].

Solving of these problems is complex and sometimes the traditional Newton-based optimization techniques may experience difficulties or even fail. A Genetic Algorithm based solution method was given in [12] to provide a comprehensive and reliable approach in determining these security conditions. More details are addressed in the following section.

4. COMPUTING THE MINIMUM DISTANCES

The problem of computing the shortest distances to the security boundaries is, in general, very complicated. In principle, it is possible to use the problem represented by equation set (3.4) to get the critical distance vector.

The closest bifurcation point is defined in the Euclidean sense. This distance in the parameter space from the operating point to the closest stability points is a good indicator for system security [17]. This index can be computed from a constrained optimization problem to minimize

$$\|\eta(x - y_0)\|$$

subject to

$$\begin{align*}
J^T(x, y_0) l' - & \alpha l' + w l' = 0 \\
J^T(x, y_0) l'' - & \alpha l'' - w l'' = 0 \\
l_i' = & 1 \\
l_i'' = & 0
\end{align*}$$

where $\alpha$ and $w$ are the real and imaginary parts of an eigenvalue of $J$ and $l' + j l''$ is the corresponding eigenvector. The last two equations are to make sure the solutions are non-trivial. Examples of utilizing this technique are given in [10]. This general method can find saddle node and Hopf bifurcations, as well as minimum and maximum damping conditions should such characteristic points exists in the ray of optimization. A singularity induced bifurcation can also be detected during building of the reduced system Jacobian. This comprehensive approach covers the important local bifurcations and other important small disturbance stability conditions in one optimization approach. The bifurcation point, not always a global one. Generally there exist multiple close bifurcation points [17]. Global ones can be located by special search algorithms such as Genetic Algorithms [12].

Ideas similar to those put forward in [6] can also be tried. For example, after obtaining the closest instability point along the given direction $\delta y$, it is possible to analyze the angle between the loading direction and left eigenvector of the appropriate eigenvalue of $J$ (state matrix) in this point. As this vector shows the normal direction with respect to the stability boundary, the angle computed can be used to rotate $\delta y$ in the direction where the distance decreases.

A more efficient way is the direct computation of the closest instability point. The following modification of the general optimization problem (3.12) and (3.13) is addressed:

$$\|y - y_0\|^2 \Rightarrow \min$$

subject to

$$\begin{align*}
f(x, y) = & 0 \\
J^T(x, y) l' - & \alpha l' + w l' = 0 \\
J^T(x, y) l'' - & \alpha l'' - w l'' = 0 \\
l_i' = & 1 \\
l_i'' = & 0
\end{align*}$$

Similar to problem (3.12), (3.13), the success of the algorithm depends upon the initial guesses of the variables and selected eigenvalues. The choice of these parameters is a complicated task. One can use some practical ideas regarding the most dangerous loading direction and critical eigenvalues (for example, corresponding to the inter-area oscillatory modes). It is quite difficult to get all of the critical and subcritical distances by this approach using traditional Newton based optimization techniques.
G\ened Algorithms (GAs) are capable of locating the globally optimal solution. GAs usually contain the following steps: generation of an initial population, evaluation of the fitness function, such genetic operations as crossover, reproduction and mutation, and forming a new generation. The procedure is repeated until some termination criterion is met and the optimum is thus obtained [18].

GAs with sharing function method can locate the global and several local optima in the search domain. Sharing function methods re-scale an individual's original fitness value (or solution candidate and objective function value in terms of the optimization problem) based on its distance in the search domain to the neighboring individuals. With this method, those individuals located in a densely populated neighborhood receive lower fitness, and those in a loose neighborhood receive full fitness value. Consequently, the algorithm iterates more steps with several solutions of similar re-scaled fitness, and the global one as well as the local optima are among them. This sharing function method can be used to comprehensively locate both the critical and subcritical distances from the current operating point toward instability limits [12]. These distances can be designed to include important directions of operation in the space of any power system parameters of interest. For example, they can be associated with such stability conditions as critical and/or sub-critical distances, the minimum damping conditions, the Saddle node or Hopf bifurcations, and load reactive power demands at buses 3 and 6 respectively; the sensitivities of reactive power generation with generators, shunt capacitors and/or SVCs. These actions all help move the system to operate at a stable equilibrium with adequate security.

In [13-15] a coordinated control technique is proposed to provide an efficient way to increase small disturbance stability margins in the power system parameter space. The technique is based on the geometric understanding of computing an optimal direction in a parameter space. Basic ideas were first proposed in [120] and further developed by Alvarado et al. in [16]. The computation of an optimum direction is based on the sensitivity of the minimum Euclidean distance to stability boundaries with respect to controlled parameters. We use a simple power system - see Figure 1 - to illustrate the application of the proposed technique.

5. Security Control

Following the computation of the stability boundaries and minimum distances, there is useful information for the design of preventive and corrective control. Given the system current operating point, the sensitivities of distance with respect to controlled parameters are computed in [13] with \( Q_{ds}, n_1, n_2, B_5, B_6, B_9, B_{12} \) being the most influential parameters to affect the security margin. Therefore, control actions shall be the combination of decreasing load demand at bus 5, tap ratio of tap changer 1 and increasing capacitor support at bus 5. It is practical to consider multiple minimum distances to get an optimal remedial control direction. Let the distance function be:

\[
F_d = \frac{1}{2} \sum_{i=1}^{n} (d_i - d_0) 
\]

Figure 1. A 6-Bus Power System
where \( d_i \) is the i-th distance and \( d_0 \) is the safe marginal distance. The sensitivity \( \frac{\partial \hat{d}_i}{\partial \tau} \) gives the optimal direction in the controlled parameter space. We give an illustrative example to investigate this from the geometric point of view in a two dimensional space. Two minimum distances \( \hat{d}_1 \) and \( \hat{d}_2 \) are selected and it is assumed that \( \hat{d}_1 < \hat{d}_2 < d_0 \). For each distance the sensitivity

\[
\frac{\partial \hat{d}_i}{\partial \tau} = \left[ \frac{\partial \hat{d}_i}{\partial \tau_1}, \frac{\partial \hat{d}_i}{\partial \tau_2} \right]
\]

gives the optimal direction in the space specified by \( \tau_1 \) and \( \tau_2 \) to decrease \( \hat{d}_i \). The direction is anti-parallel to the normal vector at the critical point \( \tau^* \) and can be expressed as [16]:

\[
\vec{a} = \frac{\partial \hat{d}_i}{\partial \tau_1} \vec{i} + \frac{\partial \hat{d}_i}{\partial \tau_2} \vec{j} \\
\vec{b} = \frac{\partial \hat{d}_2}{\partial \tau_1} \vec{i} + \frac{\partial \hat{d}_2}{\partial \tau_2} \vec{j}
\]

(5.2)

(5.3)

where \( \vec{i} \) and \( \vec{j} \) are the unit vectors for axes \( \tau_1 \) and \( \tau_2 \) respectively. The overall optimum direction \( \vec{a} \) is the linear combination of \( \vec{a} \) and \( \vec{b} \) with a multiplier of the distance mismatch \( (\hat{d}_1 - \hat{d}_2) \),

\[
\vec{o} = (\hat{d}_1 - \hat{d}_2) \vec{a} + (\hat{d}_2 - d_0) \vec{b}
\]

(5.4)

For the model power system, assuming \( \rho_0 = 2.0 \), the sensitivities are:

\[
\frac{\partial \hat{d}_1}{\partial \tau_1} = -0.8609 \\
\frac{\partial \hat{d}_2}{\partial \tau_1} = -1.1420
\]

and the optimal direction for remedial control is illustrated in Figure 2.

Figure 2. Optimal direction in a two dimensional plane

To have the optimal control direction in the controlled parameter space for power system security operation is useful, but more can be achieved. An optimal scheduling of control actions for those selected controlled parameters having a large influence on system security is possible. The authors in [13, 14] proposed a framework for such an optimal control algorithm. An optimal control direction requires a specific combination of dissimilar available controls, i.e. load control, tap control, capacitor switching, and may also include power electronic controls. The response speeds for various controls are different, varying from less than a second for a SVC to a minute or so for a tap changer. Additionally, different controls have different priorities in practical implementation. All these indicate that practical arrest of voltage collapse should be achieved via a sequence of control actions, which are selected from different types available [21]. The solution is achieved with a multiple stage constrained optimization problem, with the objective to minimize the control effort/cost or maximizing the control effect subject to control capability, system stability and quality of supply constraints. The problem can be formulated to minimize

\[
J(\tau) = \sum_{t=1}^{N} C(u_t, \tau_t), \quad u_t, \tau_t \in \mathbb{R}^n
\]

subject to

(i) controls capability constraints

\[
t_{i,low} \leq \tau_t \leq t_{i,upper}, \quad t = 1, 2, \ldots, N
\]

(ii) stability constraints

\[
S_{\text{margin}}(\tau_t) - S_{\text{margin}}(\tau_t-1) > \epsilon
\]

(5.5)

(5.8)

where \( t = 1, 2, \ldots, N \) indicates the stages in the optimization. \( C(*) \) is the selected cost function at each stage \( u_t, \tau_t \) are the state and decision variables at stage \( t \\ t_{i,low} \) and \( t_{i,upper} \) are the lower and upper bounds of \( \tau_i \) for the state variables \( \tau_i \) is the new security margin after stage \( t \) with decision \( \tau_t \) chosen and \( \epsilon \) is the assigned minimum requirement for security margin increase at each stage.

Depending on what the concern is, the cost function could account for the overall control effort, economic cost, and the total deviation of voltage profile from the reference value or some combination of them. In [13, 14], the authors use a minimum angle solution, where the angle is taken as the difference between control direction at each stage and the optimal direction - see Figure 3.

Figure 3. Minimum angle criterion
Solid line: the optimal direction; Dashed line: actual control sequence.

State variables \( u_t \) represent the set of all the controls available at stage \( t \), while decision variables \( \tau_t \),...
represent the control actions needed to be taken at stage \( t \). The target is to find a policy \((\tau_1, \tau_2, \ldots, \tau_T)\) which minimizes the cost function while subject to the constraints. Control constraints (i) relate for example to the maximum/minimum capacitance and tap ratio available and the maximum load shedding allowable. The stability constraint (ii) requires that the chosen policy satisfy a system stability condition at each stage. Dynamic programming can be applied to this multiple stage optimization problem [22]. This method requires the construction of an optimal decision at each stage for each control variables. Further illustration of the method is given in \([14, 15]\) including application to the NSW power systems.

6. CONCLUSIONS

The paper has reviewed some important concepts of small disturbance stability computation and control from a nonlinear viewpoint. Bifurcations are essential to the study of small disturbance stability sensitivities. A general framework has been described for the computation of various bifurcations, load flow feasibility and damping boundaries. From these the operating limits can be found in different directions of the parameter space. The minimum distances to the boundaries (local and global) can then be evaluated and used to compute optimal directions for security control. Finally, it has been shown how to use these directions to optimally coordinate dissimilar control actions.

REFERENCES


